Slides for
Advanced Microeconomic Analysis
(NEKN21)

Fall 2011

Preliminary

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Content

1.Introduction: General Remarks and Utility in Decision Making

Purpose of the course: Learning outcomes

Knowledge and understanding

Students shall:

• deepen their knowledge of the theory of decision making under uncertainty, game theory and industrial organisation including theories of monopoly, oligopoly and contracts,

• be able to translate economic and social interaction problems into formalised games,

• be able to describe practically encountered incentive problems in terms underlying information asymmetries and be able to analyse them,

• understand the basic theory of decision making under risk so that they are able to explain the consistency requirements, including the "independence axiom".

Applying knowledge and making judgements

Students shall have the ability to independently:

- solve monopoly pricing problems, including problems that involve second and third degree price discrimination,
- solve games using solution concepts such as dominance, Nash equilibrium and subgame perfection, and be able to derive substantive economic conclusions from the solutions,
- evaluate arguments about likely implications of different actions on the way a market works in terms of competition and price formation.

Learning skills

Students shall have a command of the course contents so that they will be able to deepen their knowledge by independently studying more advanced literature and scientific articles written at a reasonably technical level.

Prerequisites

NEKB21 (Intermediate Micro, which in turn strongly recommends NEKB22, Mathematical and Statistical Methods for Economics)

Practical Information

- Lectures (time, place)
  Focus: i) importance, ii) difficulty
  Language

- Plan of the course (see plan of the lectures)

- Literature (see plan of the lectures)

- Examination
  - Home assignment (10p, only valid in the ordinary and first re-exam)
    - rules (Deadline: September 29 at noon, Available no later than September 20)
  - Written examination (90p, no calculators)
  - Pass: 50p (that is 40p on the exam if 10p are earned in the home assignment).
  - Ordinary exam, re-exam (about one month after)
  - Re-take (written exam: 100p, points from home assignment not valid)
  - Examinations outside Lund or on other occasions will <u>not</u> be given!

Utility in Decision Making

Background:

Sketch of the Decision-making process (figure)

- There are different DM theories
- Focus on utility maximization theory (UMT)
- UMT is the dominating theory and provides economists worldwide with a common framework and language to analyse economic problems

UMT: Some basic concepts and observations

From preferences to utility:

Notation

- Consumption set: $X = R_{+}^{k}$
- Preferences: $x \geq * y$: $x = (x_1, x_2, x_3, ..., x_k)$ is "at least as good as" $y = (y_1, y_2, y_3, ..., y_k)$
- $\geq *$ -an ordering and a binary relation (over X).
- Strict preference: $x > * y$: can be defined by "not $y \geq * x$" and "not $x = * y$" (indifference).

Standard assumptions

1. Completeness

2. Reflexivity

3. Transitivity

Note: 1, 2, 3 => rational preference relation

4. Continuity (rules out discontinuity)

5. Strong monotonicity

Introduce the utility function $u : X \to R$, such that $u(x) > u(y)$ iff (if and only if) $x >^* y$.

Proposition: If 1,2,3,4,5 hold, then there exist a continuous $u : R_+^k \to R$, which represents these preferences.

Proof: (sketch, only of existence)

Utility maximization: the indirect utility function

The optimal solution: x^* is obtained from

$\max u(x)$ such that $px \leq m$.

Remarks:

- A solution, x^*, exists if $u(x)$ is continuous and $B = \{x \in X : px \leq m\}$ is closed and bounded ($p_i > 0$ for $i = 1,2,3,\ldots,k$).

If local nonsatiation (LNS) is assumed we can construct an indirect utility function (IU) by:

$v(p,m) = \max u(x)$ such that $px = m$

($px^* = m$ follows from LNS)

Properties of $v(p,m)$:

1. Non-increasing in p and non-decreasing in m.

   (Follows from that $B' \subseteq B$ for $p' \geq p$ or $m \geq m'$)

2. Homogenous of degree 0.

   (Follows from $v(tp, tm) = v(p, m)$, example.)

3. Quasiconvex in *p*.

4. Continuous.

2. Decision under Uncertainty

Content:

Different forms of uncertainty

Lotteries

The Expected utility function

Risk aversion

Different forms of uncertainty

- Risk: The outcomes and their respective probabilities are (objectively or subjectively) “known”. (Ex. insurance)

- Ambiguity: The set of outcomes are known, but probabilities are unknown or partially unknown. (taking an exam.)

- Genuine uncertainty: The set of outcomes and probabilities are unknown or partially unknown (launch of new product, some R&D projects)

=> The focus here is on risk.

Lotteries (sometimes also denoted as "*gambles*")

A risky choice can be represented by a lottery:

$p \circ x \oplus (1-p) \circ y$: means that the prizes *x* and *y* is received with probability *p* and (1-*p*), respectively. (The signs "$\circ$" and "$\oplus$" can be interpreted as general expressions that there is some relationship between terms. This is used before we have a theory of that tell us more exactly how to combine the terms. An alternative is: (*x*,*p*;*y*,1-*p*).)

Assumptions about the lottery:

L1. Certainty is equivalent to probability 1.

L2. Order does not matter.

L3. Compound lotteries can be reduced.

Expected Utility

Let L be a space of lotteries that satisfies L1-L3 and over which the agent has a rational preference ordering.

We will investigate the possibility of an expected utility function $u : L \to R$ with some desirable properties.

Existence: With the same arguments as before existence can be shown.

But,

- Any monotonic transformation of a given utility function would work.

➔ Additional assumptions needed to get a manageable expected utility function.

Expected utility property (EUP):

(Utility functions satisfying EUP is also called "*von Neumann-Morgenstern utility functions*)

$$u\left(p \circ x \oplus (1-p) \circ y\right) = pu(x) + (1-p)u(y)$$

→Utility is additively separable over outcomes and linear in probabilities.

Assumptions (to obtain EUP):

U1: Continuity

U2: Independence of combination (of prizes)

U3: Existence of best (*b*) and worst lottery (*w*)

U4: A higher probability of *b* is preferred.

Expected utility theorem:

If a preference ordering over lotteries $(L, \geq *)$ satisfies L1-L3 and U1-U4 then there is a utility function $u : L \rightarrow R$ that satisfies EUP.

Proof: Let $u(b) = 1$ and $u(w) = 0$.

1. Construction of utility function: Let the utility of an arbitrary lottery *z* be given by $u(z) = p_z$, where $p_z \circ b \oplus (1 - p_z) \circ w = *z$ ("=*":indifference.) Existence and uniqueness due to U1 and U4, respectively.
2. EUP can be checked by transforming $u(p \circ x \oplus (1 - p) \circ y)$ to $pu(x) + (1 - p)u(y)$. (verification of utility function) -Numerical example.

Uniqueness of VNM utility function:

- Ordinary utility functions: Any monotonic transformation will represent the same preferences.
- An VNM function contains more information

To see this consider lottery *x*: $p_x \circ b \oplus (1-p_x) \circ w =^* x$ → $u(x) = p_x u(b) + (1-p_x) u(w)$, where p_x is a meaningful number, representing preferences. To see this more clearly rearrange to get: $\frac{u(x)-u(w)}{u(b)-u(w)} = p_x$.

The ratio of utility differences must be the same for each VNM-function representing the same preferences → p_x is not just an ordinal number.

Possible transformations of VNM-function:

- Affine transformations: An affine transformation of $u(x)$ is given by $v(x) = au(x) + c$, where $a \geq 0$.

- Demonstration that affine transformations does not alter preferences.
- Demonstration that any monotonic transformation preserving EUP of $u(x)$ is an affine transform.

Other notations for expected utility

- $\sum_{i=1}^{n} p_i u(x_i)$, where *n* is a finite number.
- $\int u(x)p(x)dx$, where $p(x)$ is a continuous probability function.

Risk aversion (RA)

RA is normally assumed:

→ $u(p \circ x \oplus (1-p) \circ y) \leq u(px + (1-p)y)$

RA is related to the concavity of $u(w)$, but $u''(w)$ would not be a suitable measure since it is not invariant to affine transformations.

→ Arrow-Pratt (AP) measure: $r(w) = -\frac{u''(w)}{u'(w)}$.

Interpretation: The AP measure and the willingness to accept small gambles

The acceptance set, $A(w)$, at wealth *w*, is the set of acceptable lotteries with prizes (x_1, x_2) that are paid out conditional on an event (E) to occur.

The boundary of $A(w)$ is given by $x_1(x_2)$ such that

$$pu(w+x_1)+(1-p)u(w+x_2(x_1))\equiv u(w)$$

Remarks:

- The slope of the boundary at (0,0) → $x_2'(x_1)=-\frac{p}{1-p}$

(Indicates the odds at which an individual is prepared to accept a small gamble.)

- An individual *j* is at least as willing to take risk as another individual *i* if $A_i(w)\subseteq A_j(w)$. (Global risk aversion).

- An individual *j* is at least as willing to take risk as another individual *i* around (0,0) if $x_i''(0)\leq x_j''(0)$. (local risk aversion).

→ Demonstrate that $x''(0)$ will be contingent on $r(w)$.

-Local risk aversion can be and has been elicited in economic experiments (see e.g., Binswanger, 1980, Holt and Laury, 2002)

Example (from study by Holm, Opper and Nee, 2010)

R1

Decision	Option A (probabilities of payoffs)	Option B (probabilities of payoffs)	Circle your choice of option
1	10% of 300 CNY 90% of 240 CNY	10% of 580 CNY 90% of 15 CNY	A B
2	20% of 300 CNY 80% of 240 CNY	20% of 580 CNY 80% of 15 CNY	A B
3	30% of 300 CNY 70% of 240 CNY	30% of 580 CNY 70% of 15 CNY	A B
4	40% of 300 CNY 60% of 240 CNY	40% of 580 CNY 60% of 15 CNY	A B
5	50% of 300 CNY 50% of 240 CNY	50% of 580 CNY 50% of 15 CNY	A B
6	60% of 300 CNY 40% of 240 CNY	60% of 580 CNY 40% of 15 CNY	A B
7	70% of 300 CNY 30% of 240 CNY	70% of 580 CNY 30% of 15 CNY	A B
8	80% of 300 CNY 20% of 240 CNY	80% of 580 CNY 20% of 15 CNY	A B
9	90% of 300 CNY 10% of 240 CNY	90% of 580 CNY 10% of 15 CNY	A B
10	100% of 300 CNY	100% of 580 CNY	A B

Application: The demand for insurances:

Demonstration that full coverage will be demanded from a competitive insurance industry if the individual is risk averse.

Global risk aversion

It is harder to determine if person A is globally more risk averse than person B. Let $A(w)$ and $B(w)$ be A:s and B:s utility functions. (Note that $A(w)$ does not denote the acceptance set here!)

Three equivalent definitions of global risk aversion

A is (globally) more risk averse than B if:

1. $r_A(w) > r_B(w)$ for any *w*.

2. $A(w) = G(B(w))$, where $G(\cdot)$ is a strictly concave function. $\Leftrightarrow A(w)$ is a strictly concave transformation of $B(w)$.

3. $\pi_A(\tilde{\varepsilon}) > \pi_B(\tilde{\varepsilon})$, where $\pi_i(\tilde{\varepsilon})$ is the risk premium to avoid the lottery with outcome $w + \tilde{\varepsilon}$ and with $E\tilde{\varepsilon} = 0$.

(Formally, for A: $A(w - \pi_A(\tilde{\varepsilon})) = EA(w + \tilde{\varepsilon})$)

Pratt's theorem: 1-3 are equivalent.

Proof (Only 1→2 and 2→3):

1→2: Differentiate $A(w) = G(B(w))$ wrt to *w* twice and manipulate to obtain expressions for $r_A(w), r_B(w)$.

2→3: Start from $A(w - \pi_A(\tilde{\varepsilon})) = EA(w + \tilde{\varepsilon}) = EG(B(w + \tilde{\varepsilon}))$ and use definitions and Jensen's inequality to obtain $\pi_A(\tilde{\varepsilon}) > \pi_B(\tilde{\varepsilon})$.

Some comments on:

- Relative risk aversion
- State dependent utility
- Subjective probabilities
- Behavioral violations of expected utility

3. Monopoly

Content:

Some basics under uniform pricing

Price discrimination

- 1st degree price discrimination
- 2nd degree price discrimination
- 3rd degree price discrimination

Some basics under uniform pricing

Profit maximization problem under perfect competition

vs. monopoly: *p* as given or *p*(y) as a function.

Monopoly: → FOC: $p(y)+p'(y)y=c'(y)$

Remarks:

i) FOC→ $p(y)>c'(y)$ → inefficient

ii) FOC→ $p(y)>r'(y)=MR$ (different from perf. comp.)

iii) FOC→ $p(y)\left(1+\frac{1}{\varepsilon(y)}\right)=c'(y)$ → the monopolist will

produce at the elastic part of the demand curve.

The inverse demand curve: a comment

The monopolist faces:

$\max_{p,y}\ py-c(y)$ such that $D(p)\geq y$

since $D(p)=y$ is reasonable to assume →

$$\max_p \; pD(p) - cD(p).$$

However, is often more convenient to work with the inverse: $y = D^{-1}(y)$.

→ $\max_y \; p(y)y - c(y)$.

Special Cases:

- Linear inverse demand and constant MC=*c*

→ $p(y) = a - by$

→ $y^* = \frac{a-c}{2b}, p^* = \frac{a+c}{2}$

- Constant elasticity of demand:

$\varepsilon(y) = -b$ → $p^* = \frac{c}{1 - \frac{1}{b}}$

- Comparative Statics:

General remarks: Suppose $f(x)$ is to be optimised and we know that the optimisation is affected by parameter *a* so we might write: $f(x(a),a)$.

→ FOC: $\frac{\delta f(x(a),a)}{\delta x} \equiv 0$, by differentiation both sides wrt to *a* → $\frac{dx}{da} = -\frac{\frac{\delta^2 f(x(a),a)}{\delta x \delta a}}{\frac{\delta^2 f(x(a),a)}{\delta x^2}}$

A similar reasoning can be applied to $\pi(y(c),c)$ where MC=c→

→ $\frac{dy}{dc} = -\frac{\frac{\delta^2 \pi}{\delta y \delta c}}{\frac{\delta^2 \pi}{\delta y^2}} \leq 0$

Price Discrimination (P.d.)

Background:

Assume: Quasilinear utility function:

$U(x,y)=u(x)+y$, where $u(0)=0$ and y is interpreted as money.

Let $r(x)$ be the (total) maximum willingness to pay (WTP) for consumption level x. → $u(x)\equiv r(x)$.

→ Marginal WTP $u'(x)=p(x)$ - the inverse demand curve.

1st degree price discrimination

The price charged for each unit is equal to the consumers' maximal WTP.

Problems: Separation and Arbitrage.

The monopolist's problem:

$\max_{r,x} r - cx$ such that $u(x) \geq r$, where r is revenue.

→ FOC: $u'(x*) = c$.

i) $u'(x*) = c$ → Pareto efficiency.

ii) $u(x*) = r$ → the monopolist extracts all surplus.

iii) same result can be obtained if it is assumed that each unit is sold separately.

2nd degree price discrimination

Price differs depending on the number of units bought, but not across consumers → self-selection.

Model to derive some general results:

Assume: Two consumer types, where

$U_i(x_i, y_i) = u_i(x_i) + y_i$ for $i = 1,2$ and furthermore that

$u_1(x_1) < u_2(x_2)$ and $u_1'(x_1) < u_2'(x_2)$ for all x_1 and x_2.

- Monopolist will choose $p(x)$ (a nonlinear function to maximize profits).

- Each consumer spends $p(x_i)x_i = r_i$.

→ Four constraints out of which two must bind.

By manipulations it can be shown that $p(x)$ will be chosen so that $r_2 = u_2(x_2) - u_2(x_1) + r_1$ and $r_1 = u_1(x_1)$.

→ The high demand consumer is indifferent between the two consumption levels.

→The low demand consumer pays his maximum WTP.

It can also be demonstrated that the high demand consumer gets the efficient amount, i.e., $u'_2(x_2) = c$.

- Graphical illustration.

3rd degree price discrimination

Different consumer groups are charged different prices, but each consumer faces a constant price for all units.

Assume: Two separate markets, constant MC →

Monopolist's problem:

$\max\ p_1(x_1)x_1 - p_2(x_2)x_2 - c(x_1 + x_2)$

FOC → $p_1(x_1^*) > p_2(x_2^*)$ iff $|\varepsilon_1| < |\varepsilon_2|$

→ The price in the elastic market is lowest.

Welfare effects:

Let $u(x_1, x_2) + y$, be an aggregate utility function for the two groups, where $u(x_1, x_2)$ is concave. Furthemore, let $c(x_1, x_2)$ be the cost of producing (x_1, x_2).

→ Social Welfare: $W(x_1, x_2) = u(x_1, x_2) - c(x_1, x_2)$.

Consider a change from uniform pricing $\left(p_1^0 = p_2^0 = p^0\right)$ to price discrimination $\left(p_1^{'}, p_2^{'}\right)$:

By concavity and by some manipulations, the following inequality must apply for the welfare change:

$$\left(p^0 - c\right)\left(\Delta x_1 + \Delta x_2\right) \geq \Delta W \geq \left(p_1^{'} - c\right)\Delta x_1 + \left(p_2^{'} - c\right)\Delta x_2$$

→ i) A necessary condition for $\Delta W > 0$ is that the quantity increases $\Delta x_1 + \Delta x_2 > 0$.

→ii) A sufficient condition for $\Delta W > 0$ is that the weighted output change is positive,

$\left(p_1^{'} - c\right)\Delta x_1 + \left(p_2^{'} - c\right)\Delta x_2 > 0$.

5. Oligopoly Theory

Contents

Cournot competition

(Extensive reading: From Stability of the system + 16.2, p.287-9.)

Duopoly

Players: Firm 1 and 2.

Strategies: $y_i \geq 0$.

Payoff: $\pi_i = p(Y)y_i - c_i(y_i)$, where $y_1 + y_2 = Y$.

A NE in this game is a pair (y_1^*, y_2^*) such that each firm maximizes profit given the other firm's choice.

Characterization of NE:

1st order condition: $\frac{\partial \pi_i}{\partial y_i} = p(Y) + p'(Y)y_i - c_i'(y_i) = 0$.

2nd order condition: $\frac{\partial^2 \pi_i}{\partial y_i^2} = p''(Y)y_i + 2p'(Y) - c_i''(y_i) \leq 0$.

Reaction curve (firm 1): $f_1(y_2)$ – firm 1:s optimal

response (in terms of quantity) to each choice of firm 2.

--> $\frac{\partial \pi_1 (f_1(y_2), y_2)}{\partial y_1} \equiv 0$

Interpretation: $f_1(y_2)$ specifies 1:s optimal choices (i.e., where F.O.C. is satisfied) for various beliefs about y_2.

The slope of $f_1(y_2)$ can be analyzed by differentiating the identity above w.r.t. y_2 :--> $\frac{\partial \left(\frac{\partial \pi_1 (f_1(y_2), y_2)}{\partial y_1} \right)}{\partial y_2} = 0$

--> $f_1'(y_2) = -\frac{\frac{\partial^2 \pi_1}{\partial y_1 \partial y_2}}{\frac{\partial^2 \pi_1}{\partial {y_1}^2}}$

Note, $\frac{\partial^2 \pi_1}{\partial y_1^{\ 2}} \leq 0$

$$\frac{\partial \frac{\partial \pi_1}{\partial y_1}}{\partial y_2} = p'(Y) + p''(Y) y_1$$

---> If the inverse demand curve, $P(Y)$ is concave or not

too convex --> $\frac{\partial^2 \pi_1}{\partial y_1 \partial y_2} < 0$ --> $f_1'(y_2) < 0$.

If $f_1'(y_2) < 0$ --> (y_1, y_2) are strategic substitutes.

If $f_1'(y_2) > 0$ --> (y_1, y_2) are strategic complements.

Example: Linear demand and constant marginal cost.

Characterization of NE when there are several firms

Start from 1st order condition:

$$\frac{\partial \pi_i}{\partial y_i} = p(Y) + p'(Y) y_i - c_i'(y_i) = 0$$ -->

$p(Y) + p'(Y)y_i = c_i'(y_i)$, rearrange and use $s_i = \frac{y_i}{Y}$,

$$\varepsilon = \frac{dY}{dp}\frac{p}{Y} \leftrightarrow \frac{1}{\varepsilon} = \frac{dp}{dY}\frac{Y}{p} \text{ -->}$$

$p(Y)\left[1 + \frac{s_i}{\varepsilon}\right] = c_i'(y_i)$, where s_i is firm *i*:s market share and ε is the (market) demand price elasticity.

If there are *n* identical firms with constant marginal costs, *c*. →

$$p(Y)\left[1 + \frac{1}{n\varepsilon}\right] = c.$$

Remark: $p(Y) \to c$ as $n \to \infty$.

Bertrand Competition

Duopoly model

Players: Firm 1 and 2.

Strategies: $p_i \geq 0$.

Payoff for 1: $\pi_1 = (p_1 - c_1) d_1(p_1, p_2)$, where c_1 is (constant) marginal cost and

$$d_1(p_1, p_2) = \begin{cases} D(p_1) \leftarrow p_1 < p_2 \\ D(p_1)/2 \leftarrow p_1 = p_2 \\ 0 \leftarrow p_1 > p_2 \end{cases}$$

Payoff for 2: Symmetrically defined.

A NE is a pair (p_1^*, p_2^*) such that each firm maximizes profit given the other firm's choice.

Case: $c_1 = c_2$

NE is given by $p_1^* = p_2^* = c_1 = c_2$.

Intuition + best responses (Figure)

Case: $c_1 < c_2$

NE is given by $p_1^* = c_2(-\varepsilon), p_2^* = c_2$.

Intuition

Case: $c_1 < c_2$ and fixed costs

NE is given by $p_1^* = c_2 \leq p_2^*$ if $\pi_1(c_2, p_2^*) \geq 0$.

Otherwise → PNE may not exist.

Model with sales under Bertrand competition

Players: Firm 1 and 2.

Strategies: $F(p)$ - cumulative df, which gives the probability that a price is below *p*.

r – reservation price, where $F(r)=1$.

Consider symmetric equilibra <-> players use the same strategy → if 1 choses *p* and 2 plays $F(p)$, then

- $F(p)$ is the probability player 2 has the lowest price
- $1-F(p)$ is the probability 1 has the lowest price).

Payoff: Depends on demand from *I* (informed) customers who shop where the price is lowest and *U* customers who is loyal to their respective firm.

$$d_1(p_1, p_2) = \begin{cases} U + I \leftarrow p_1 < p_2 \\ U \leftarrow p_1 > p_2 \end{cases}$$

Expected profit at a given price when the opponent plays $F(p)$

$\pi = p(1-F(p))(I+U)+pF(p)U-k$, where *k* is a fixed cost (assume zero marginal costs).

→ Expected profit when a distribution of prices is chosen as a strategy:

$$\bar{\pi} = \int_0^{\infty}[p(1-F(p))(I+U)+pF(p)U-k]f(p)dp$$

(the probability that *p* is chosen will be given by $f(p)$)

Derivation of equilibrium

For all prices *p* in the equilibrium distribution (such that $f(p)>0$) the expected profit must be the same and equal to $\bar{\pi}$. --> at a given price *p*:

$\bar{\pi} = p(1-F(p))(I+U)+pF(p)U-k$ -->

$$F(p)=\frac{p(I+U)+-k-\overline{\pi}}{pI}$$

Determine $\overline{\pi}$ at $F(r)=1$--> $\overline{\pi}=rU-k$.

--> $F(p)=1-\frac{U}{I}\left(\frac{r}{p}-1\right)$

Complements and Substitutes (Cournot and Bertrand)

Distinguish between firm 1 and 2:s markets:

Inverse demand functions:

$$p_1 = \alpha_1 - \beta_1 y_1 - \gamma y_2 \, , \; p_2 = \alpha_2 - \gamma y_1 - \beta_2 y_2$$

Direct demand functions:

$$y_1 = a_1 - b_1 p_1 + c p_2 \, , \; y_2 = a_2 + c p_1 - b_2 p_2$$

Remarks:

1. Condition for perfect substitutes: $\alpha_1 = \alpha_2$ and

   $\beta_1 = \beta_2 = \gamma$.

2. Condition for independence: $\gamma = 0$

3. Index for product differentiation: $\gamma^2 / \beta_1 \beta_2$

--> Independence $\rightarrow \gamma^2 / \beta_1 \beta_2 = 0$

--> Perfect substitutes: $\rightarrow \gamma^2 / \beta_1 \beta_2 = 1$

4. If proper substitution → Cournot and Bertrand have the same mathematical structure.

Cournot: $\max_{y_1} [\alpha_1 - \beta_1 y_1 - \gamma y_2] y_1$

Bertrand: $\max_{p_1} [a_1 - b_1 p_1 + c p_2] p_1$

5. Reaction curves. The Bertrand reaction curve can be obtained by substitution from the Cournot best response function.

$y_1 = \dfrac{\alpha_1 - \gamma y_2}{2\beta_1}$ Change greek letters to roman -->

$$p_1 = \frac{a_1 + c p_2}{2b_1}.$$

Strategies are normally strategic substitutes in Cournot competition and strategic complements in Bertrand competition. (Figure).

Stackelberg competition

Players: Firm 1 and 2.

Strategies: $y_1 \geq 0$, $y_2 \geq 0$.

Payoff: $\pi_i = p(Y)y_i - c_i(y_i)$, $i = 1,2$.

Information: 2 observes y_1 before choosing y_2 → Sequential game, where $y_2 = f_2(y_1)$. Firm 1 is called the leader and 2 the follower.

Solution: Start by 2:s problem:

1st order condition: $p'(Y)y_2 + p(Y) - c_2'(y_2) = 0$ (same as in Cournot)

→ $y_2 = f_2(y_1)$

1:s problem :

$$\max_{y_1} \pi_1 = p(y_1 + f_2(y_1))y_1 - c_1(y_1)$$

1st order condition:

$$p'(Y)y_1 + p'(Y)f_2'(y_1)y_1 + p(Y) - c_1'(y_1) = 0$$

Remarks:

1. The second term expresses that 1 takes into account that increasing output will decrease market prices, but when output is increased 2 will react and change her output too (normally by decreasing output). As a consequence this second term is usually positive.
2. Firm 1 will choose the optimal point on firm 2:s reaction curve (illustration) → Profit for leader is higher than in Cournot competition.

Example: Linear demand, identical constant marginal costs.

Relative profit levels: Under some relatively weak assumptions it can be shown that the leader has higher profit than the follower.

A1: $\pi_1(y_1, y_2)$ is a strictly decreasing function in y_2 and $\pi_2(y_1, y_2)$ is a strictly decreasing function in y_1.

A2: $f_1(y_2)$ and $f_2(y_1)$ are strictly decreasing functions.

(*Proof, extensive reading*)

Price leadership

Players: Firm 1 and 2.

Information: 2 observes p_1 before choosing p_2 →

Sequential game, where $p_2 = g_2(p_1)$. Firm 1 is called

price leader.

Strategies: $p_1 \geq 0$, $p_2 = g_2(p_1) \geq 0$.

Payoff: $\pi_1 = p_1 x_1(p_1, g_2(p_1)) - c_1(x_1(p_1, g_2(p_1)))$ and

$\pi_2 = p_2 x_2(p_1, p_2) - c_2(x_2(p_1, p_2))$.

1st order conditions

Follower: $\frac{\partial \pi_2}{\partial p_2} = p_2 \frac{\partial x_2}{\partial p_2} + x_2(p_1, p_2) - c_2'(x_2)\frac{\partial x_2}{\partial p_2} = 0$

Leader:

$$\frac{\partial \pi_1}{\partial p_1} = p_2 \left[\frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2} g_2'(p_1) \right] + x_1(p_1, p_2) -$$

$$- c_1'(x_1) \left[\frac{\partial x_1}{\partial p_1} - \frac{\partial x_1}{\partial p_2} g_2'(p_1) \right] = 0$$

Slope of the reaction function:

$g_2'(p_1)$ is solved for after differentiation of the best

response function $\frac{\partial \pi_2}{\partial p_2} \equiv 0$ (in the case when $c_2'(x_2) = 0$)

$$\frac{\partial \frac{\partial \pi_2}{\partial p_2}}{\partial p_1} = p_2 \frac{\partial x_2}{\partial p_2} \left[\frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial p_2} g_2'(p_1) \right] + \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial p_2} g_2'(p_1) = 0$$

$$g_2'(p_1) \left[p_2 \frac{\partial^2 x_2}{\partial p_2^2} + \frac{\partial x_2}{\partial p_2} \right] = - \left[p_2 \frac{\partial^2 x_2}{\partial p_1 \partial p_2} + \frac{\partial x_2}{\partial p_1} \right]$$

$$\rightarrow g_2'(p_1) = \frac{-\left(p_2 \frac{\partial^2 x_2}{\partial p_2 \partial p_1} + \frac{\partial x_2}{\partial p_1}\right)}{p_2 \frac{\partial^2 x_2}{\partial p_2^2} + \frac{\partial x_2}{\partial p_2}}.$$

Denominator negative from 2nd order condition → sign will be determined by the numerator. The numerator's last term is positive for substitutes. → Reaction curves normally upward sloping in this model.

Example: Perfect substitutes: $p_1 = p_2$

Follower takes p_1 as given and supply $S_2(p_1)$.

Leader will maximize

$\pi_1 = p_1 r(p_1) - c_1(r(p_1))$, where $r(p_1) = x_1(p_1) - S_2(p_1)$

--> FOC: $r(p_1) + p_1 r'(p_1) - c_1'(r(p_1)) r'(p_1) = 0$

(Monopolist's problem facing $r(p_1)$:Illustrate

Collusion

If the firms start to cooperate, then it is reasonable to expect that they try to maximize their joint profit →

$$\max_{y_1, y_2} \pi = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2)$$

1st order conditions

$$\frac{\partial \pi}{\partial y_1} = p(y_1^* + y_2^*) + p'(y_1^* + y_2^*)(y_1^* + y_2^*) - c_1'(y_1^*) = 0$$

$$\frac{\partial \pi}{\partial y_2} = p(y_1^* + y_2^*) + p'(y_1^* + y_2^*)(y_1^* + y_2^*) - c_2'(y_2^*) = 0$$

Remark

1. $c_1'(y_1^*) = c_2'(y_2^*)$.
2. Each firm has incentives to deviate, since at (y_1^*, y_2^*) it must be the case that $\dfrac{\partial \pi_i(y_1^*, y_2^*)}{\partial y_i} > 0$, where

$\pi_i(y_1, y_2) = p(y_1 + y_2)y_i - c_i(y_i)$.

To prove this, the effect of an deviation from the

collusive outcome $\left(y_1^*, y_2^*\right)$ on firm 1:s profit is given by

$$\frac{\partial \pi_1\left(y_1^*, y_2^*\right)}{\partial y_1} = p\left(y_1^* + y_2^*\right) + p'\left(y_1^* + y_2^*\right)y_1^* - c_1'\left(y_1^*\right) = -p'\left(y_1^* + y_2^*\right)y_2^*$$

The RHS follows from the 1st order condition (above)

and since $p'\left(y_1^* + y_2^*\right) < 0$ we know that $\frac{\partial \pi_1\left(y_1^*, y_2^*\right)}{\partial y_1} > 0$.

Conjectural Variation

Different beliefs about the other firm's responses can be used to understand the difference between the models. (However, these conjectures should not be taken literally in these one-shot games.) Let v_{12} denote 1:s conjecture how 2 responds to 1:s choice of output. Then the 1st order condition (for firm 1) can be written as:

$$p(Y) + p'(Y)(1 + v_{12})y_1 - c_1'(y_1) = 0.$$

Cases:

Cournot: If $v_{12} = 0$.

Perfect competition: $v_{12} = -1$.

Stackelberg: $v_{12} = f_2'(y_1)$.

Collusive "equilibrium": $v_{12} = \frac{y_2}{y_1}$.

Remark: If the effect on profit of a small change in 2s output (dy_2) in a collusive equilibrium where 1 has threaten to respond by $dy_1 = v_{21} dy_2 = y_2 / y_1$ is studied, it can be shown that such an expected response stabilizes the cartel.

Repeated oligopoly games

The oligopoly situations described so far can be repeated and analyzed as repeated games → may increase realism, but less tractable to analyze and usually larger equilibrium sets.

Illustration: Finitely repeated Cournot game.

NE may exist, where the firms cooperate.

Consider a two-period game, where 1 uses the following punishment strategy:

In the 1st period set $y_1 = y_1^*$. If in the first period $y_2 = y_2^*$ (where (y_1^*, y_2^*) can e.g., be the cartel outputs), then set $y_1 = y_1^C$ in the 2nd period, otherwise choose a y_1 such that $\pi_2(y_1, \cdot) \leq 0$.

→ A necessary condition for 2 to cooperate will be:

$$\pi_2\left(y_1^*, y_2^*\right) + \delta\pi_2\left(y_1^C, y_2^C\right) > \max_{y_2} \pi_2\left(y_1^*, y_2\right).$$

If this condition is satisfied for both players and both use such a similar punishment strategy → NE that support cooperation may exist. However, cooperation builds on non-credible threats→ The unique SPE is repeated play of the one-shot Cournot equilibrium.

Infinitely repeated oligopoly games

If the time horizon is infinite collusion can be supported by SPE both in Bertrand and Cournot competition. (Same logic as in an infinitely repeated prisoner's dilemma.)

Illustration: Cournot competition

Definitions:

π_i^* - one-period cartel profit for firm *i*.

π_i^C - one-period Cournot profit for firm *i*.

π_i^d - one-period profit for firm *i*, if *i* maximizes profit given *j*:s choice of y_j^*.

Punishment strategy (played by both players): Choose y_i^* at $t = \tau$ if $y_j^{t-1} = y_j^*$ otherwise set $y_i^t = y_i^C$ for $t \geq \tau$.

It is then optimal for *i* to keep the cartel output if:

$$\frac{\pi_i^*}{1-\delta} > \pi_i^d + \frac{\delta \pi_i^C}{1-\delta} \Rightarrow \delta > \frac{\pi_i^d - \pi_i^C}{\pi_i^* - \pi_i^C}.$$

Remarks: i) Different sequences of quantity choices than the ones optimal for a cartel can be supported by SPE. ii) Other punishment strategies (than the grim "punish forever") can support SPE.

Entry Decisions

Entry Pattern

Two firms choose an entry time, *t*.

If the firm is the only entrant at *t* → Monopoly profit $\pi_1(t)$.

If at *t* two firms have entered → Duopoly profit $\pi_2(t)$, where $\pi_1(t) > \pi_2(t)$.

Illustrate in figure.

Remarks: i) NE and SPE may differ. ii) All monopoly profit is dissipated in SPE.

Limit Pricing

Potential competition may restrict firms' prices on an existing market. However, this is not always the case and to show that such a behavior can be supported in equilibrium is not trivial.

Example when potential competition has no effect.

Example when potential entry can have an effect.

Bertrand duopoly and uncetainty about incumbent’s cost

→ Game of incomplete information.

(If time: Illustrate in a game tree).

6. Information

Contents

Background and Basics

Hidden Action

Hidden Information

(Note: Extensive reading in Varian: 25.4 (graphical analysis); 25.5)

Background and Basics

Principal – agent problems

- P:s payoff is contingent on A:s agent

- P:s problem: To carefully design contract that is optimal given constraints in terms of information and the A:s optimising behaviour.

- Examples: Insurance contracts, 2nd degree price discrimination, Owner – manager contracts (bonuses).

Variants:

  - Hidden action: A:s action is not perfectly observable by P.

  - Hidden information: A:s objective function is not perfectly observable by P.

Hidden action

Monopoly

Full information model: Some general remarks

Standard Model:

- Finite number of output levels: $(x_1,...,x_n)$

- A can take action *a* or *b*, which affect of the probability of the outcomes.

- π_{ij} -probability for outcome x_i if action *j* is taken.

- $s(x_i)$ payment to A if x_i is observed.

- P:s payoff and A:s payoff.

Analysis:

Suppose P wants A to take action b.

→ (1) Incentive compatibility constraint (IC)

→ (2) Participation constraint (PC)

Dilemma for P: P wants to pay A less when output is low, but cannot be sure if the low output is due to low effort (action a) or bad luck.

Optimisation Problem:

$$V(b) = \max_{s_i} \sum_{i=1}^{n} (x_i - s_i)\pi_{ib}$$

such that $\sum_{i=1}^{n} u(s_i)\pi_{ib} - c_b \geq \bar{u}$

$$\sum_{i=1}^{n} u(s_i)\pi_{ib} - c_b \geq \sum_{i=1}^{n} u(s_i)\pi_{ia} - c_a$$

Form the Lagrange function → FOC →

$$\frac{1}{u'(s_i)} = \lambda + \mu\left(1 - \frac{\pi_{ia}}{\pi_{ib}}\right)$$

Remarks:

1. PC is generally binding → $\lambda > 0$.

2. If $\mu = 0$ → Payment independent of outcome → $c_a > c_b$: A and P has common interest → Problem reduced to an insurance problem.

3. If $\mu > 0$ → Payment dependent of outcome and P wants b but $c_a < c_b$ → Conflict of interest.

4. If $\mu > 0$: P:s optimal contract will be based on statistical inference, namely how likely it is that he has chosen P: preferred action (b).

If $\frac{\pi_{ia}}{\pi_{ib}}$ (the likelihood ratio) is decreasing in x_i → $s(x_i)$ will be a monotone function of x_i.

5. Comparative statics

Analyze how P:s payoff is contingent c_a and c_b.

Example with mean-variance utility

Competitive market

A few remarks:

Competition → many Ps → P:s profit = 0.

→ PC not binding

Two cases:

Full insurance solution → A gets a fixed amount regardless of effort

Partial insurance solution → A:s payment contingent on output.

Example: Moral Hazard (equilibrium may not exist)

Hidden information

Monopoly

Assumption: Single crossing property (SCP)

Model: Two agent types, two actions

- P:s optimization problem:

$$\max_{x_1, x_2, s_1, s_2} (x_1 - s_1)\pi_1 + (x_2 - s_2)\pi_2$$

subject to PC and SSC constraints.

- Analysis of binding constraints

- Characterization of optimal solution

(Compare to 2^{nd} degree price discrimination)

Competition: Market equilibrium

Zero profit condition

Pooling vs. separating equilibrium

Hidden information: Specific Mechanisms

Adverse selection

- The lemons market

Signaling

- Warranties
- Education

7. Summing Up

Advanced Microeconomics $\approx$ to learn a language

Involves learning:

I.) Concepts (glossary)

II.) How concepts are related (synonyms, grammar)

III.) General economic insights by combinations of concepts in models. (speaking)

Examples:

I. Concepts:

EUP, Global risk-aversion, SPE, Self-selection constraint, Strategic substitutes, SCP.

II. How concept are related:

1. If a preference ordering satisfies a number of assumptions → there is a EUP utility function

2. Three global notions of risk aversion are synonymous (Pratt's theorem)

3. For a monopoly the first-order condition that $MR(y) = c'(y)$ is equivalent to $p(y)\left(1 + \frac{1}{\varepsilon(y)}\right) = c'(y)$

4. The shape of the demand curve is related to the slope of the reaction function in Cournot competition.

III. General insights

Combination of concepts normally involve the following components

Assumptions

- Motivation (utility, profits)
- Choice variables (consumption, prices)
- Information (complete, hidden)
- Situation (individual, strategic)

Optimisation

→ Economic insights

Example 1: Cournot

Motivation: profit

Choice variable: quantities

Situation: strategic interaction among firms producing perfect substitutes with identical and constant marginal costs.

Information: complete

Optimisation: given others' choices (Nash)

Economic insight: Equilibrium price decreases in the number of firms (from monopoly price to marginal cost).

Example 2: Hidden action: Principal-Agent

Motivation: utility (payoffs)

Choice variable: payment schedule, action

Information: P does not observe action

Situation: distributions of outcomes satisfy the monotone likelihood property, $C_a < C_b$.

Optimisation: P:s optimises given some restrictions regarding A.

Economic insight: Optimal contract payments will be contingent upon statistical inferences about the likelihood that A has chosen the desired action.