Sealed Bid Auctions vs. Ascending Bid Auctions: An Experimental Study *

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Abstract

This paper considers the sealed bid and ascending auction, which both identifies the minimum Walrasian equilibrium prices and where truthful preference revelation constitutes an equilibrium. Even though these auction formats share many theoretical properties, there are behavioral aspects that are not easily captured. To explore this issue in more detail, this paper experimentally investigates what role the design of the auction format has for its outcome. The results suggest that the sealed bid mechanism performs weakly better in all of investigated measures (consistent reporting, efficiency etc.). In addition, we find that the performance of the ascending auction is increasing over time, whereas the sealed bid auction shows no such tendency.

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1 Introduction

Auctions are common practice, when allocating and pricing scarce resources, on a variety of markets. Examples include the markets for spectrum licences, debts, emissions and commodities (e.g. fish, wool, timber etc.). The insight that the Vickrey–Clarke–Groves (VCG) sealed bid auction generates an ex post efficient outcome that in addition provides bidders with the incentives to truthfully reveal their preferences has motivated a substantial amount of research.

In particular, Demange and Gale (1985) and Leonard (1983) consider a VCG multi-item auction that identifies the minimum Walrasian equilibrium price vector and use it as a

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mechanism to allocate the items among the bidders. In such auctions, it is a weakly dominant strategy for the bidders to report their true valuations given that they wish to acquire at most one item (unit–demand bidders). A competing auction format is the ascending bid auction, where the multi-item auction from Demange et al. (1986) is a prominent example. Under this format the unit–demand bidders gradually reveal information about their demand sets until the mechanism converges. Also this mechanism identifies the minimum Walrasian equilibrium price vector and truthful bidding is an ex post equilibrium given a set of simple rules that guarantee a specific structure on the bids (see de Vries et al., 2007; Mishra and Parkes, 2007; and the more detailed discussion in Section 2.2).

Because the multi-item auctions in both the sealed bid and the ascending format identify the minimum Walrasian equilibrium price vector and have the property that truthful bidding constitutes an equilibrium they are theoretically equivalent in terms of predictions. However, there are many behavioral aspects of these two formats that are not easily captured. In particular, they differ in terms of e.g. how much information that is revealed about the bidders' valuations, how complex the format is and how information about other bidders' behavior is transmitted (see Cramton, 1998 for a discussion). Ex ante it is not evident if and how these aspects affect the outcome of the auction formats. Hence, it is natural to take the theory to data. Because we are not aware of any "real world" situation where the two auction formats are conducted in comparable contexts, the evaluation is best done by way of an economic experiment. Moreover, the experimental method enables us to have a more strict control over valuations, which is pivotal for the theoretical predictions.

To investigate what role the design of the auction format has for its outcome, we conducted an experiment with the sealed bid (Demange and Gale, 1985; Leonard, 1983) and the ascending (Demange et al., 1986) auction format as treatments. Overall we find that the sealed bid mechanism performs weakly better in the measures reported here (i.e., consistent reporting, efficiency and assignment of items). The results also confirm previous findings that subjects typically report non-truthful.[1] Moreover, for all investigated measures, there is a significantly positive time trend in the ascending treatment but not in the sealed bid treatment.

The remaining part of the paper is organized as follows. Section 2 introduces the two auction formats. Section 3 describes the experimental design and implementation. Section 4 contains the main results. Section 5 concludes the paper.

2 Auction Formats

Let the set of bidders and items be denoted by $B = \{1, ..., n\}$ and $I = \{1, ..., m\}$, respectively. Each item $i \in I$ has a price $p_i \geq 0$ where without loss of generality $p_i = 0$ represents the sellers' reservation price. The prices are gathered in the vector $p = (p_1, ..., p_m)$. The private value of item $i \in I$ to bidder $b \in B$ is represented by v_{bi}. Consequently, each bidder $b \in B$ is characterized by a vector of type $v_b = (v_{b1}, ..., v_{bm})$. There is also an

[1] See e.g. Attiyeh et al. (2000), Harstad (2000), Kagel et al. (1987), Kawagoe and Mori (2001) and Kagel and Levin (1993) for similar results.

unlimited number of "no items" (denoted by 0) whose prices and values always equal zero. The demand set for bidder $b \in B$ at prices p is defined by:

$$D_b(p) = \{i \in I \cup \{0\} : v_{bi} - p_i \geq v_{bj} - p_j \text{ for all } j \in I \cup \{0\}\}.$$

A price vector p is said to be a Walrasian equilibrium price vector if there is an assignment $x : B \mapsto I$ such that $x_b \in D_b(p)$ for all $b \in B$ and $x_b \neq x_{b'}$ if $b' \neq b$ and $\{x_b, x_{b'}\} \subseteq I$, i.e., each bidder is assigned an item from his demand set and each item different from the "no item" can be assigned to at most one bidder. The pair (p, x) is a Walrasian equilibrium if p is a Walrasian equilibrium price vector and if $x_b \neq i$ for all $b \in B$ then $p_i = 0$, i.e., if an item is not assigned to any bidder, then its price must equal reservation price. As demonstrated by Shapley and Shubik (1972) the set of Walrasian equilibrium price vectors is non-empty and forms a complete lattice. Thus, the existence of a unique minimum Walrasian equilibrium price vector $p^{\min}$ is guaranteed.

2.1 The Sealed Bid Format

In the sealed bid mechanism (Demange and Gale, 1985; Leonard, 1986) each bidder $b \in B$ reports his values of the items $\hat{v}_b$ to the auctioneer. This report may be truthful or not but based on it the auctioneer identifies the minimum Walrasian equilibrium price vector $\hat{p}^{\min}$ by solving a simple LP-problem. Given that $\hat{p}^{\min}$ is used as a mechanism to allocate the items among the bidders, the sealed bid auction has a (weakly) dominant strategy equilibrium where each bidder $b \in B$ reports $\hat{v}_b = v_b$. This results in an ex post efficient outcome in the sense that the sum $\sum_{b \in B} v_{bx_b}$ is maximized.

2.2 The Ascending Format

The ascending bid auction (Demange et al., 1986) differs from the sealed bid auction in the sense that bidders do not submit a report of type $\hat{v}_b$ as in the latter. Instead, bidders gradually reveal information about their demand sets, for given price vectors, until the ascending mechanism converges. To formalize the procedure, let the set of bidders demanding only items in the set $S \subseteq I$ at prices p be denoted by $C(S, p) = \{b \in B : D_b(p) \subseteq S\}$. A set of items S is said to be overdemanded if the number of bidders demanding only items in this set is greater than the number of items in the set, i.e., if $|S| < |C(S, p)|$. An overdemanded set with the property that none of its proper subsets is overdemanded is called a minimal overdemanded set.

The ascending mechanism can be described as follows. The auctioneer announces a price vector p. Each bidder then reports (truthful or not) his demand set $\hat{D}(p)$.[2] If there is no over demanded set of items, the mechanism terminates. Otherwise, prices are increased for an arbitrary minimal overdemanded set of items according to some rule.[3] The procedure

[2] If this set contains more than one item, the bidder is indifferent between all reported items.

[3] Such a rule may be to increase the price of all items in the selected minimal overdemanded set S by one unit (Demange et al., 1986) or to ask the agents that only demand items from the set S to report the

is repeated until the family of overdemanded sets is empty. As demonstrated by Demange et al. (1986), this procedure will always identify an efficient minimum Walrasian price equilibria in a finite number of iterations given that reports are truthful in each iteration. In addition, truthful reports constitute an equilibrium given that bids are consistent with some activity rule[4] (Vries et al., 2007; Mishra and Parkes, 2007).

3 Experimental Design and Implementation

The experiment was conducted at Lund University in September 2010. We ran four separate sessions consisting either of a sealed bid- (S) or an ascending (A) multi-item auction treatment. In total 117 subjects participated (60 in treatment S and 57 in treatment A). The subjects were students at the introductory or intermediate level in Economics at Lund University. Instructions were given both written and aloud to the subjects at the beginning of the experiment.[5] We also conducted a test period in order for the subjects to familiarize with the software.

At the beginning of the session, subjects were randomly assigned to a bidder type (1, 2 or 3) and a group consisting of three subjects of different type. Each three-person group was fixed in all 10 periods of the session. All subjects knew that they were grouped with two other participants, but could not discern who they were.

The subjects were informed that they would participate in an auction over three items (denoted by 1, 2 and 3). In particular, they were given the information that if they were awarded item 1, 2 or 3, then (i) they have to pay a price and (ii) the item will automatically be resold at the end of the period at a predetermined price called the resale value. The resale values v_b of bidder type $b \in \{1, 2, 3\}$ for items $i \in \{1, 2, 3\}$ was given by:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{bmatrix} = \begin{bmatrix} 20 & 20 & 100 \\ 20 & 60 & 100 \\ 40 & 60 & 80 \end{bmatrix} \tag{1}$$

The resale value of the ”no item” was zero. Each subject knew his/her resale values but not the resale values of the other two participants. Truthful reports under both treatment S and A given (1) yields the assignment $(x_1, x_2, x_3) = (3, 2, 1)$ with corresponding minimum Walrasian equilibrium prices $(p_1, p_2, p_3) = (0, 20, 60)$, i.e., bidder type 1 is assigned item $x_1 = 3$ at price $p_3 = 60$ and so on.

The subjects in treatments S and A received the following information of how the mechanism worked:

minimum price increase that would make them indifferent to an item outside S (Andersson and Andersson, 2011). This paper adopts the latter rule.

[4]The meaning of an activity rule is that bidders not are allowed to submit conflicting reports across the iterations of the ascending procedure. For example, if bidder b reveals that $\hat{v}_{bi} - \hat{v}_{bi'} \geq \delta$ for some $\delta \in \mathbb{N}$ in iteration t, then it cannot be the case that the very same bidder reveals that $\hat{v}_{bi} - \hat{v}_{bi'} < \delta$ in some subsequent iteration $t + k$.

[5]A transcript of the instructions is available at http://www.nek.lu.se/NEKTAN/AuctionExperiment.htm.

"The prices for items 1, 2 and 3 will be determined automatically by a computer program according to a predetermined rule which is based on the reports of all three members of the group."

As previously explained in Section 2, these reports was given by a vector of type $\hat{v}_b$ in treatment S and the reported demand sets in each iteration of the ascending mechanism under treatment A. Similarly, the only information given to the subjects regarding which item they will be assigned was the following treatment equivalent information:

"Each group member will be awarded the item where the difference between the stated valuation and the calculated price is the highest." [Treatment S]

"If each group member can be assigned an item from his/her reported demand set, the auction is terminated" [Treatment A]

The payoff in each period was given by the difference between the resale value of the item assigned to the subject and its price. If the subject was assigned "no item", the payoff was zero. At the end of the experiment, the accumulated payoffs were converted into Swedish kronor according to an exchange rate of 1 experimental currency unit = 0.5 SEK.[6] Subjects received a show-up fee of 50 SEK and the average earnings were 314 SEK. A session took approximately 45 minutes to conduct.

4 Results

We start by an analyzing the of subjects' reporting behavior.[7,8] Following Olson and Porter (1994) we define a measure of consistent reporting as follows: In the S treatment we say that a report is *consistent* if the order of $\hat{v}_b$ preserves the order of v_b for subject b. For example, if the subject has valuation profile 1 then we require that $\hat{v}_{13} > \hat{v}_{12} = \hat{v}_{11}$ for the report $\hat{v}$ to be consistent. In the A treatment we say that a report is *consistent* if $\hat{D}_b(p) = D_b(p)$ where $\hat{D}_b(p)$ and $D_b(p)$ are the reported and the true demand set for subject b at prices p, respectively. Subjects that report inconsistent and consistent are assigned the values 0 and 1, respectively, in the consistency measure. As a consequence, the average consistency measure for a sample of subjects always belongs to the closed interval $[0, 1]$.

The average consistency measures for the two treatments are reported in columns 2 and 3 of Table 1. A first observation is that the average consistency measure is well below the equilibrium prediction (i.e. 1.00), which should not come as a surprise given the findings of previous experimental papers (see footnote 1). A second observation is that the S treatment has a higher degree of consistency in every period. To facilitate a statistical

[6]At the time of experiment 1 SEK $\approx$ 0.11 EUR.

[7]In order to facilitate a comparison between treatments S and A we only analyze the final iteration in A.

[8]For the interested reader Table 5 in the Appendix reports the average prices by period and treatment for each item.

Table 1: Mean values for the investigated measures.

	Consistency		Equilibria		Efficiency	
Period	A	S	A	S	A	S
1	0.19	0.53	0.11	0.60	0.69	0.96
2	0.18	0.48	0.26	0.45	0.81	0.94
3	0.33	0.57	0.37	0.65	0.86	0.95
4	0.39	0.63	0.32	0.60	0.89	0.95
5	0.35	0.58	0.42	0.45	0.90	0.93
6	0.37	0.60	0.42	0.25	0.91	0.85
7	0.33	0.48	0.42	0.45	0.89	0.93
8	0.40	0.55	0.47	0.50	0.92	0.94
9	0.41	0.53	0.37	0.45	0.92	0.93
10	0.37	0.57	0.53	0.55	0.94	0.95
All periods	0.33	0.55	0.37	0.50	0.87	0.93
Period 1–5	0.29	0.56	0.29	0.55	0.83	0.94
Period 6–10	0.38	0.55	0.44	0.44	0.91	0.92

comparison between the two treatments the average consistency within each three-person group was calculated. In this way 39 independent observations (20 in treatment S and 19 in treatment A) were created. Using a two–sided Mann–Whitney test we find that there is a statistical difference between the treatments in overall mean consistency (p-value = 0.000). To examine the effects of experience the sample was divided into two categories: Subjects are defined to be *inexperienced* if they are in period 1–5 and *experienced* otherwise. Again the two–sided Mann–Whitney test reveals that there is a significant difference in consistency for inexperienced (p-value = 0.000) as well as for experienced (p-value = 0.002) subjects.

Even if the S treatment has a significantly higher degree of consistency it is evident by studying Table 1 that the gap between the treatments is smaller in later periods indicating that there is a possible positive time trend in the data for treatment A. To investigate this closer we estimated a linear random effects regression with the consistency measure as the independent variable and including a time variable as the dependent variable.[9] Interestingly, the regression estimates in Table 2 reveal that there is a positive time trend in the A treatment but not in the S treatment.

We next report the fraction of groups in a selected sample with an assignment of items according to the equilibrium prediction. This will also be a measure in the closed interval $[0, 1]$. By adopting the same statistical test as before we found that there is a significant difference between the two treatments when subjects are inexperienced (p-value = 0.011) but not when they are experienced (p-value = 0.954) or overall (p-value = 0.126). Our next observation is that exactly as for the consistency measure, it is clear (from columns 4

[9]Since we are not interested in making forecasts we use linear regressions even though they might give predictions outside $[0, 1]$ interval. Corresponding probit estimations reveal the same patters as reported here.

Table 2: Estimations on consistency

	Treatment A	Treatment S
Period	0.0198***	0.0008
	[0.0031]	[0.0078]
Constant	0.221***	0.549***
	[0.0283]	[0.0480]
Observations	570	600

Robust standard errors in brackets. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

and 5 of Table 1) that treatment S is closer to the theoretical prediction in the first periods whereas this gap is closed in later rounds. Thus, one can expect that there is a positive time trend not only for the consistency measure but also for the equilibrium measure. To investigate this in more detail we estimated a linear random effects regression using the group allocation dummy as the dependent variable and including a time variable as an independent variable. The regression output in Table 3 verifies the above suspicion that there is a strong positive time trend in the A treatment but not in the S treatment.

Table 3: Estimations on equilibrium outcomes

Treatment	A	S
Period	0.0198**	0.0024
	[0.0092]	[0.0091]
Constant	0.523***	0.707***
	[0.1020]	[0.0848]
Observations	190	200

Robust standard errors in brackets. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

It is also interesting to see how close, in terms of efficiency, each group was to the equilibrium prediction. In line with Olson and Porter (1994) we calculate an efficiency index for each three-person group as follows:

$$E = \frac{\sum_b \sum_i y_{bi} v_{bi}}{\sum_b \sum_i y^*_{bi} v_{bi}}, \tag{2}$$

where y_{bi} is an indicator variable taking the value 1 if subject b is assigned item i and zero otherwise, and y^*_{bi} is the indicator variable given true reports (i.e. $y^*_{13} = y^*_{22} = y^*_{31} = 1$ and $y^*_{bi} = 0$ for the remaining pairs (b, i)). Because the sum of valuations is maximized when reports are consistent (see Section 2) the denominator in (2) will always be weakly larger than the numerator. Hence, also this measure produces a number in the closed interval $[0, 1]$.

Columns 6 and 7 in Table 1 show the mean efficiency by period in the S and A treatment. The efficiency measure is quite similar across treatments, with only a slight advantage for the static mechanism in early periods. To facilitate a statistical analysis we adopt the two-sided Mann–Whitney test as in the above. The test demonstrates that there is a statistical advantage for the static mechanism when subjects are inexperienced (p-value = 0.000) but not experienced (p-value = 0.6907). Overall there seems to be a advantage for the static mechanism (p-value = 0.001). To investigate if there is a positive time trend in treatment A also for this measure we adopt the the same methodology as for the equilibrium measure and estimate a random effects panel regression. The regression output in Table 4 reveals that there is a significant positive time trend in the A treatment but not in the S treatment.

Table 4: Efficiency estimations

Treatment	A	S
Period	0.0199***	-0.0018
	[0.0034]	[0.0016]
Constant	0.762***	0.941***
	[0.0340]	[0.0113]
Observations	190	200

Robust standard errors in brackets. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

5 Conclusion

In summary we find that the sealed bid mechanism dominates the ascending counterpart in all measures reported here. The difference between the two auction formats is more pronounced when subjects are inexperienced than when subjects are experienced. This latter finding is further strengthen by the presence of significant positive time trend in the ascending treatment measures. No time trend is not found in the sealed-bid treatment. It is therefore alluring to conclude that after a suitable number of periods, the ascending bid mechanism will dominate the sealed bid counterpart. One should however always be careful when extrapolating results and further experimental results are needed to validate this claim. In regards to the policy dimension it is risky to draw general conclusions from a single experiment and we are in general unwilling to do so. But we think that it is safe to say that if an auction is to be conducted just once, then the sealed bid mechanism is to prefer.

Appendix

This Appendix reports the mean prices by period and treatment (see Table 5). We also recall that the minimum Walrasian equilibrium prices in our experimental setting is given by the price vector $(p_1, p_2, p_3) = (0, 20, 60)$.

Table 5: Mean prices

	Price item 1		Price item 2		Price item 3	
Period	A	S	A	S	A	S
1	0.32	0.75	2.26	8.90	16.26	31.05
2	0.05	0.00	1.58	5.60	2.160	25.90
3	1.58	0.00	3.58	10.7	13.16	32.15
4	0.00	0.00	5.84	7.75	16.58	30.65
5	0.00	0.00	2.84	6.20	18.84	36.75
6	0.26	0.00	4.05	8.70	25.00	33.40
7	0.05	0.00	9.00	2.85	24.79	24.75
8	0.11	0.00	2.53	8.60	24.42	33.25
9	0.00	0.00	4.53	6.40	14.95	30.90
10	0.00	0.00	4.26	6.70	20.32	34.40
All periods	0.24	0.08	4.05	7.24	17.65	31.32
Period 1-5	0.39	0.15	3.22	7.83	13.40	31.30
Period 6-10	0.08	0.00	4.87	6.65	21.89	31.34

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